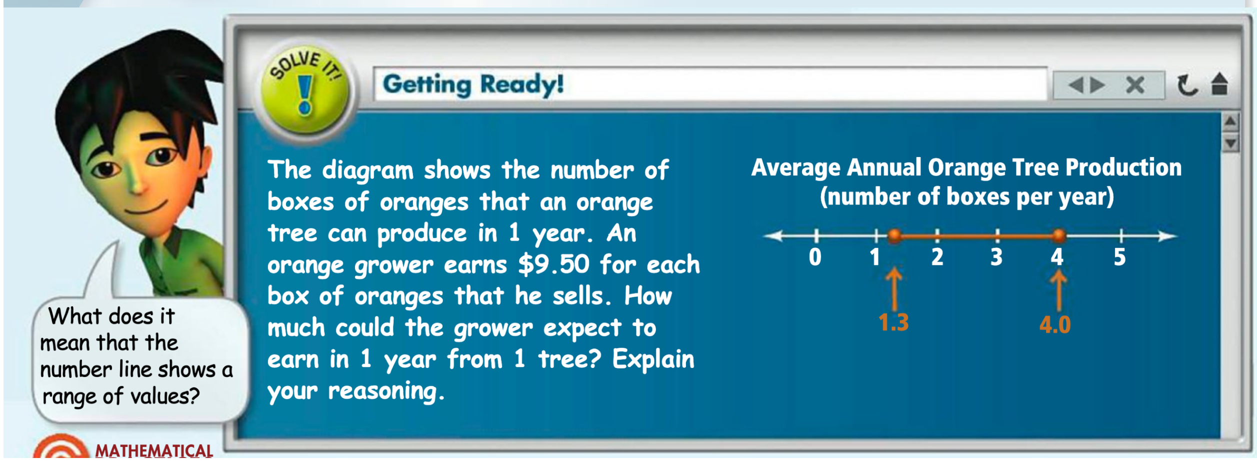
3-6 Compound Inequalities

and , or



Let e=earnings

$e \geq 9.50(1.3)$

and

$e \leq 9.50(4)$

$e \geq 12.35$ and

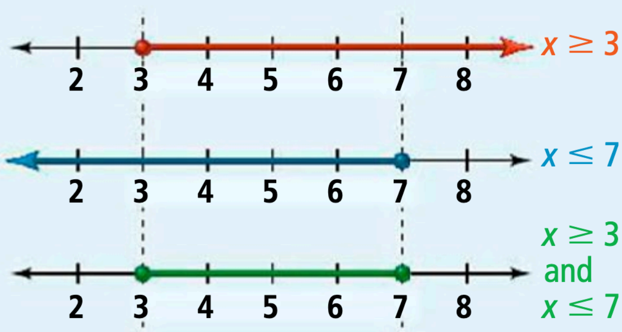
$e \leq 38$

between \$12.35 and \$38

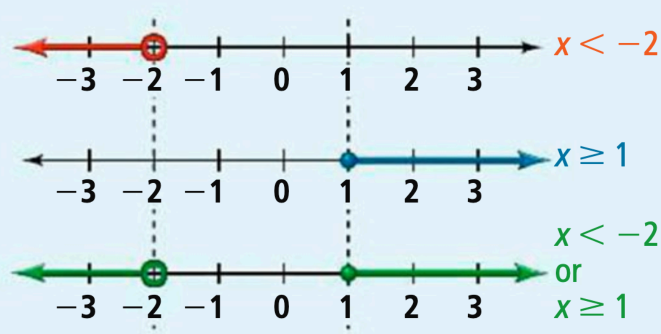
The Solve It problem involves a value that is between two numbers. You can use a compound inequality to represent the relationship. A ***compound inequality*** *consists of two distinct inequalities joined by the word* *and* *or the word* *or*.

You find the solutions of a compound inequality either by identifying where the solution sets of the distinct inequalities overlap or by combining the solution sets to form a larger solution set.

The graph of a compound inequality with the word *and* contains the *overlap* of the graphs of the two inequalities that form the compound inequality.



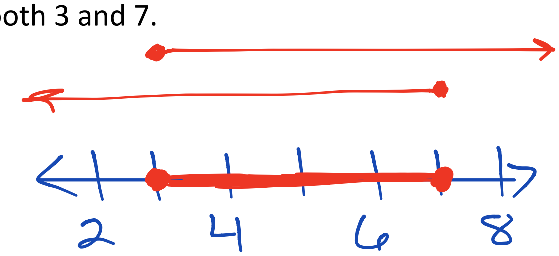
The graph of a compound inequality with the word *or* contains *each* graph of the two inequalities that form the compound inequality.



You can rewrite a compound inequality involving *and* as a single inequality. For instance, in the inequality above, you can write $x \geq 3$ and $x \leq 7$ as $3 \leq x \leq 7$. You read this as "*x* is greater than or equal to 3 and less than or equal to 7." Another way to read it is "*x* is between 3 and 7, inclusive." In this example, *inclusive* means the solutions of the inequality include both 3 and 7.

$x \geq 3$ and $x \leq 7$

$3 \leq x \leq 7$



$3 < x < 7$

$3 \leq x < 7$

PROBLEM 1: WRITING A COMPOUND INEQUALITY

What compound inequality represents each phrase? Graph the solutions.

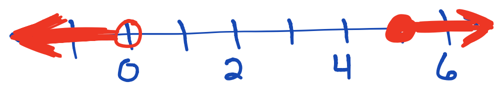
a) all real numbers that are greater than -2 $\boxed{\text{and}}$ less than 6.

$x > -2$ and $x < 6$

$-2 < x < 6$

b) all real numbers that are less than 0 or greater than or equal to 5

$x < 0$ or $x \geq 5$



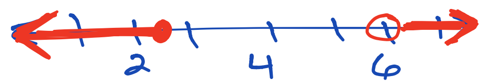
c) all real numbers that are greater than or equal to -4 $\boxed{\text{and}}$ less than 6

$x \geq -4$ and $x < 6$

$-4 \leq x < 6$

d) all real numbers that are less than or equal to 2 ½ $\boxed{\text{or}}$ greater than 6

$x \leq 2\frac{1}{2}$ or $x > 6$



e) all real numbers that are between -5 and 7

$x > -5$ and $x < 7$

$-5 < x < 7$

f) The circumference of a women's basketball must be between 28.5 in and 29 in, inclusive

$x \geq 28.5$ and $x \leq 29$

$28.5 \leq x \leq 29$

A solution of a compound inequality involving *and* is any number that makes *both* inequalities true. One way you can solve a compound inequality is by separating it into two inequalities.

PROBLEM 2: SOLVING A COMPOUND INEQUALITY INVOLVING *AND*

Solve each inequality. Graph the solutions.

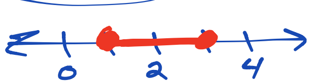
a) $\boxed{-3 \leq \boxed{m-4} \leq -1}$

$m - 4 \geq -3$ and $m - 4 \leq -1$

$+4 \quad +4 \qquad +4 \quad +4$

$m \geq 1$ and $m \leq 3$

$1 \leq m \leq 3$



b) $-2 < 3y - 4 < 14$

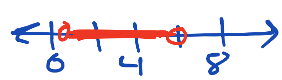
$3y - 4 > -2$ and $3y - 4 < 14$

$+4 \quad +4 \qquad +4 \quad +4$

$\frac{3y}{3} > \frac{2}{3} \qquad \frac{3y}{3} < \frac{18}{3}$

$y > \frac{2}{3}$ and $y < 6$

$\frac{2}{3} < y < 6$

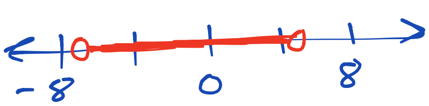


You can also solve an inequality like the ones above by working all three parts of the inequality at the same time. You work to isolate the variable between the inequality symbols.

c) $-4 < k + 3 < 8$

$-3 \quad -3 \quad -3$

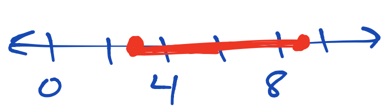
$-7 < k < 5$



d) $5 \leq y + 2 \leq 11$

$-2 \quad -2 \quad -2$

$3 \leq y \leq 9$



PROBLEM 4: SOLVING A COMPOUND INEQUALITY INVOLVING *OR*

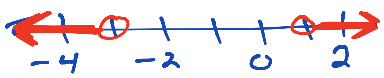
Solve each inequality. Graph the solutions.

a) $3t + 2 < -7 \ or - 4t + 5 < 1$

$-2 \quad -2 \qquad -5 \quad -5$

$\frac{3t}{3} < \frac{-9}{3} \qquad \frac{-4t}{-4} < \frac{-4}{-4}$

$t < -3$ or $t > 1$



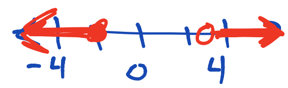
b) $-2y + 7 < 1 \ or \ 4y + 3 \leq -5$

$-7 \quad -7 \qquad -3 \quad -3$

$\frac{-2y}{-2} < \frac{-6}{-2} \qquad \frac{4y}{4} \leq \frac{-8}{4}$

$y > 3$ or $y \leq -2$

$y \leq -2$ or $y > 3$



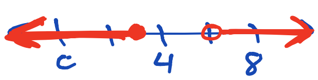
c) $7 - c < 1 \ or \ \frac{4c}{4} \leq \frac{12}{4}$

$-7 \quad -7$

$\frac{-c}{-1} < \frac{-6}{-1}$

$c > 6$ or $c \leq 3$

$c \leq 3$ or $c > 6$



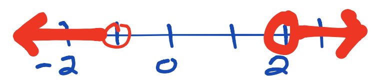
d) $5z - 3 > 7 \ or \ 4z - 6 < -10$

$+3 \quad +3 \qquad +6 \quad +6$

$\frac{5z}{5} > \frac{10}{5} \qquad \frac{4z}{4} < \frac{-4}{4}$

$z > 2$ or $z < -1$

$z < -1$ or $z > 2$

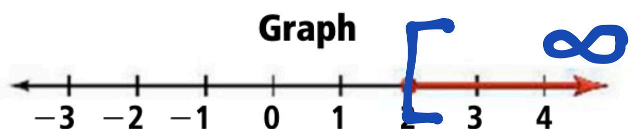
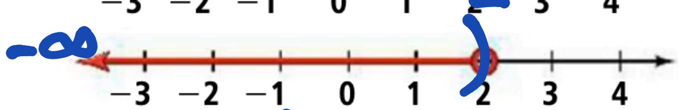
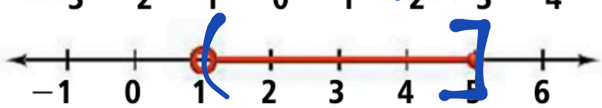
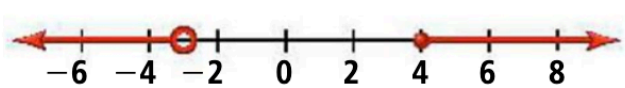


You can use an inequality such as $x \leq -3$ to describe a portion of the number line called an *interval*. You can also use *interval notation* to describe an interval on the number line. ***Interval notation*** *includes the use of three special symbols.* These symbols include:

Parentheses: Use (or) when a < or > symbol indicates that interval's endpoints are *not* included

Brackets: Use [or] when a ≤ or ≥ symbol indicates that the interval's endpoints *are* included

Infinity: Use ∞ when the interval continues forever in a *positive* direction
Use −∞ when the interval continues forever in a *negative* direction

Inequality	Graph	Interval Notation
$x \geq 2$		$[2, \infty)$
$x < 2$		$(-\infty, 2)$
$1 < x \leq 5$		$(1, 5]$
$x < -3$ or $x \geq 4$		$(-\infty, -3)$ or $[4, \infty)$

PROBLEM 5: USING INTERVAL NOTATION

Write each interval as an inequality. Graph the solutions.

a) [-4, 6)

$-4 \le x < 6$

b) (-2, 7]

$-2 < x \le 7$

c) $(-\infty, 2]$

$x \le 2$

d) [-4, 5]

$-4 \le x \le 5$

e) $[6, \infty)$

$x \ge 6$

f) $(-\infty, -1]\ or\ (3, \infty)$

$x \le -1$ or $x > 3$

Write each inequality in interval notation.

g) $x \le -1\ or\ x > 2$

$(-\infty, -1]$ or $(2, \infty)$

h) $y > 7$

$(7, \infty)$

i) $-3 \le x < 4$

$[-3, 4)$

j) $x > -2$

$(-2, \infty)$

k) $x < -2\ or\ x \ge 1$

$(-\infty, -2)$ or $[1, \infty)$

l) $-5 \le x \le 5$

$[-5, 5]$

PROBLEM 3: WRITING AND SOLVING A COMPOUND INEQUALITY

a) To earn a B in his biology course, Stan must achieve a test average between 84 and 86, inclusive. He scored 86, 85, and 80 on the first three tests of the marking period. What possible scores can he earn on the fourth test to earn a B in the course?

Let x = score on 4th test

$84 \le Avg \le 86$

$$84 \le \frac{86 + 85 + 80 + x}{4} \le 86$$

$$4(84) \le 4\left(\frac{251 + x}{4}\right) \le (86)4$$

$$336 \le 251 + x \le 344$$

$$-251 \quad -251 \quad -251$$

$$85 \le x \le 93$$

Between 85 and 93, inclusive

b) Suppose Steve scored a 78, 78, and 79 on his first three tests. Is it possible for him to earn a B in the course? Assume that 100 is the maximum grade that he can earn in the course and on the test.

$$84 \le \frac{78+78+79+x}{4} \le 86$$

$$4(84) \le \cancel{4}\left(\frac{235+x}{\cancel{4}}\right) \le (86)4$$

$$336 \le 235+x \le 344$$
$$-235 \quad -235 \quad -235$$

$$101 \le x \le 109$$

Can't do it

c) The acidity of the water in a swimming pool is considered normal if the average of three pH readings is between 7.2 and 7.8, inclusive. The first two readings for a swimming pool are 7.4 and 7.9. What possible values for the third reading *p* will make the average pH normal?

d) The County Water Department charges a monthly administration fee of \$10.40 plus \$0.0059 for each gallon of water used, up to, but not including, 7500 gal. What are the minimum and maximum numbers of gallons of water used by customers whose monthly charge is at least \$35 but no more than \$50? Express amounts to the nearest gallon.

Let g = gallons

$$35 \le \text{pay} \le 50$$

$$35 \le 10.40 + .0059g \le 50$$
$$-10.40 \quad -10.40 \quad -10.40$$

$$\frac{24.6}{.0059} \le \frac{.0059g}{.0059} \le \frac{39.60}{.0059}$$

$$4169 \le g \le 6712$$

Between 4169 and 6712 gallons, inclusive

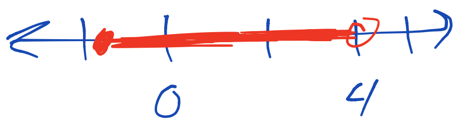
Problem 2

e) $-4 \le 2y-1 < 7$

$+1 \quad +1 \quad +1$

$$\frac{-3}{2} \le \frac{2y}{2} < \frac{8}{2}$$

$$-\frac{3}{2} \le y < 4$$



f) $-3 < -2x+5 \le 14$

$-5 \quad -5 \quad -5$

$$\frac{-8}{-2} < \frac{-2x}{-2} \le \frac{9}{-2}$$

$$4 > x \ge -\frac{9}{2}$$

$$\boxed{-\frac{9}{2} \le x < 4}$$